

根号を含む式の計算 1 解答編

I. 次の計算をせよ。

$$\begin{aligned} \textcircled{1} \quad & \sqrt{5}(\sqrt{5}-2) \\ & = 5 - 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad & -\sqrt{3}(-3+\sqrt{3}) \\ & = 3\sqrt{3} - 3 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & \sqrt{2}(1-\sqrt{2}) \\ & = \sqrt{2} - 2 \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad & \sqrt{3}(\sqrt{2}-\sqrt{48}) \\ & = \sqrt{3}\sqrt{2} - \sqrt{3} \times 4\sqrt{3} \\ & = \sqrt{6} - 12 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad & \sqrt{5}(\sqrt{20}-1) \\ & = \sqrt{5}\sqrt{2}\sqrt{2}\sqrt{5} - \sqrt{5} \\ & = 10 - \sqrt{5} \end{aligned}$$

$$\begin{aligned} \textcircled{8} \quad & \sqrt{5}(2\sqrt{3}-\sqrt{80}) \\ & = \sqrt{5} \times 2\sqrt{3} - \sqrt{5} \times 4\sqrt{5} \\ & = 2\sqrt{15} - 20 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad & \sqrt{5}(\sqrt{45}-3) \\ & = \sqrt{5}\sqrt{3}\sqrt{3}\sqrt{5} - 3\sqrt{5} \\ & = 15 - 3\sqrt{5} \end{aligned}$$

$$\begin{aligned} \textcircled{9} \quad & \sqrt{5}(1-\sqrt{5})+2\sqrt{5} \\ & = \sqrt{5} - 5 + 2\sqrt{5} \\ & = -5 + 3\sqrt{5} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad & -\sqrt{5}(1-\sqrt{5}) \\ & = -\sqrt{5} + 5 \end{aligned}$$

$$\begin{aligned} \textcircled{10} \quad & 3(\sqrt{2}-1)-\sqrt{2} \\ & = 3\sqrt{2} - 3 - \sqrt{2} \\ & = -3 + 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} \textcircled{11} \quad & \sqrt{6}(4\sqrt{30}-\sqrt{75}) \\ & = \sqrt{6} \times 4\sqrt{5}\sqrt{6} - \sqrt{6} \times 5\sqrt{3} \\ & = 24\sqrt{5} - 5\sqrt{2}\sqrt{3}\sqrt{3} \\ & = 24\sqrt{5} - 15\sqrt{2} \end{aligned}$$

$$\begin{aligned} \textcircled{12} \quad & \sqrt{2}(\sqrt{6}+3)-\sqrt{3}(\sqrt{6}+1) \\ & = \sqrt{2}\sqrt{2}\sqrt{3} + 3\sqrt{2} - \sqrt{3}\sqrt{2}\sqrt{3} - \sqrt{3} \\ & = 2\sqrt{3} + 3\sqrt{2} - 3\sqrt{2} - \sqrt{3} \\ & = \sqrt{3} \end{aligned}$$

$$\begin{aligned} \textcircled{13} \quad & \frac{\sqrt{5}}{\sqrt{8}} \left(\sqrt{32} + \sqrt{40} - \frac{1}{\sqrt{2}} \right) \\ & = \frac{\sqrt{5} \times \sqrt{4}\sqrt{8}}{\sqrt{8}} + \frac{\sqrt{5} \times \sqrt{5}\sqrt{8}}{\sqrt{8}} - \frac{\sqrt{5}}{\sqrt{8}\sqrt{2}} \\ & = 2\sqrt{5} + 5 - \frac{\sqrt{5}}{4} = 5 + \frac{8\sqrt{5}}{4} - \frac{\sqrt{5}}{4} = 5 + \frac{7\sqrt{5}}{4} \end{aligned}$$

$$\begin{aligned} \textcircled{14} \quad & \sqrt{\frac{3}{2}} + \sqrt{3} \left(\frac{1}{\sqrt{2}} - \sqrt{18} \right) \\ & = \frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}} - \sqrt{3} \times \sqrt{3}\sqrt{6} \\ & = \frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2} - 3\sqrt{6} = \sqrt{6} - 3\sqrt{6} = -2\sqrt{6} \end{aligned}$$

根号を含む式の計算 2 解答編

I. 次の計算をせよ。

- ① $(\sqrt{5} - \sqrt{3})^2$
 $= (\sqrt{5})^2 - 2 \times \sqrt{3} \times \sqrt{5} + (\sqrt{3})^2$
 $= 5 - 2\sqrt{15} + 3$
 $= 8 - 2\sqrt{15}$
- ② $(\sqrt{6} - 3)(\sqrt{6} + 3)$
 $= (\sqrt{6})^2 - 3^2$
 $= 6 - 9 = -3$
- ③ $(\sqrt{3} - 3\sqrt{5})^2$
 $= (\sqrt{3})^2 - 2 \times 3\sqrt{5} \times \sqrt{3} + (3\sqrt{5})^2$
 $= 3 - 6\sqrt{15} + 45$
 $= 48 - 6\sqrt{15}$
- ④ $(\sqrt{7} - 2\sqrt{3})(\sqrt{7} + 3\sqrt{3})$
 $= \sqrt{7}\sqrt{7} + \sqrt{7} \times 3\sqrt{3} - 2\sqrt{3}\sqrt{7} - 2\sqrt{3} \times 3\sqrt{3}$
 $= 7 + 3\sqrt{21} - 2\sqrt{21} - 6 \times 3$
 $= -11 + \sqrt{21}$
- ⑤ $(\sqrt{2} - 1)(\sqrt{2} + 2) - \sqrt{3} \cdot 2$
 $= \sqrt{2} \times \sqrt{2} + 2\sqrt{2} - \sqrt{2} - 2 - 4\sqrt{2}$
 $= 2 + 2\sqrt{2} - \sqrt{2} - 2 - 4\sqrt{2}$
 $= -3\sqrt{2}$

II. 次の各問に答えよ。

① $a = \sqrt{12}$, $b = -\sqrt{2}$ のとき
 $(a - b)^2 + 2ab$ の値を求めよ。
 $(a - b)^2 + 2ab$
 $= a^2 - 2ab + b^2 + 2ab$
 $= a^2 + b^2$
 $= (\sqrt{12})^2 + (-\sqrt{2})^2$
 $= 12 + 2$
 $= 14$

② $a = 3 - \sqrt{18}$ のとき、
 $a^2 - 6a + 9$ の値を求めよ。
 $= (a - 3)^2$
 $= (3 - \sqrt{18} - 3)^2$
 $= (\sqrt{18})^2$
 $= 18$

③ $x + y = 4$, $x - y = \sqrt{12}$ のとき、
 xy および $x^2 + y^2$ の値を求めよ。

$x + y = 4$, $x - y = \sqrt{12} = 2\sqrt{3}$ を連立方程式として解く。

$x = 2 + \sqrt{3}$, $y = 2 - \sqrt{3}$ となることから、

$$xy = (2 + \sqrt{3})(2 - \sqrt{3}) = 2^2 - (\sqrt{3})^2 = 4 - 3 = 1$$

$$x^2 + y^2 = (2 + \sqrt{3})^2 + (2 - \sqrt{3})^2$$

$$= 4 + 4\sqrt{3} + 3 + 4 - 4\sqrt{3} + 3 = 14$$

別解 $(x + y)^2 = x^2 + 2xy + y^2$ であることから

$$4^2 = x^2 + 2 \times 1 + y^2$$

$$16 = x^2 + 2 + y^2$$

$$x^2 + y^2 = 16 - 2 = 14$$